Cont-Bouchaud percolation model including Tobin tax

Gudrun Ehrenstein[1]
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Institute for Theoretical Physics,Cologne University,50923 Köln
Euroland

Abstract:
The Tobin tax is an often discussed method to tame speculation and get a source of income. The discussion is especially heated when the financial markets are in crisis. In this article we refer to foreign exchange markets. The Tobin tax should be a small international tax affecting all currency transactions and thus consequently reducing the destabilizing speculations. In this way this tax should take over a control function. By including Tobin tax in the microscopic model of Cont and Bouchaud one finds that Tobin tax could be the right method to control foreign exchange operations and get a good source of income.

1 Introduction

The reader who is familiar with the economically background of foreign exchange markets may skip this section.

1.1 Financial markets in particular the foreign exchange markets

The financial markets are composed of credit markets, security markets and foreign exchange markets. These three markets interact. In this article we re-

[1]email:ge@thp.uni-koeln.de

fer to foreign exchange markets. On foreign exchange markets currencies are traded. Exchange rate changes are a measure for the economical efficiency of national economics. The rate of exchange is the price for a foreign currency. Increasing or decreasing prices of the currencies affect the development of the particular national economy. So a devaluation causes expensive imports and low priced exports.

1.2 The system of Bretton Woods

The foreign exchange market we know today existed not before the seventies in this way. Fourty years ago, the system of Bretton Woods reigned over the international financial world. In 1944 agents of 44 nations met in Bretton Woods in the US-state New Hampshire under leadership of the victor nations to decide about the economic future after the World War II. The members of the conference brought about the World Bank and the International Monetary Fund. The arrangement of Bretton Woods was enacted in December 1945. The purpose of this meeting was to reorganize and to stabilize world commerce and international trade after the Second World War. The new world currency system is based on the warranty of best possible free convertibility of the currencies with fixed foreign exchange rates. The member states had to trade their currencies either for parities in gold or parities to the US Dollar, which showed a gold parity itself. So the US Dollar became the new world's leading currency [1].

1.3 Foreign exchange markets after Bretton Woods

The system of Bretton Woods was exempted at first from international financial crises. In Germany we know the time of economic growth as "economic miracle". The arrangement of Bretton Woods was only profitable for industrial nations. For developing countries the promised wealth was never reached. Later, the weakening strength of the US Dollar compared to European currencies lead to speculative crises.

Thus since 1971 the arrangement was abolished step by step. For reduction of the great nonequilibrium in the balances of payments between the EG currencies and the US currency, free rates of exchange were introduced

in 1973. The fixed foreign exchange rates were replaced by free fluctuating foreign exchange rates between the leading currencies which are today US Dollar, Euro and Yen. The rate of exchange system was regulated by the free game of the market forces, e.g.the private finance companies. The government did not interfere in foreign exchange market regularly. The prices for different currencies result from the difference between demand and supply. Important for the rate of exchange is the faith of the investor in the currency. Since controls for the turnover of capitals no longer exist and computers render it possible to do transactions very quickly, rates of exchanges are determined by short-term financing. Consequently, the foreign exchange markets are not a true measure for the economical efficiency of the nations [2].

1.4 Where does the trade with foreign exchanges take place ?

62% of the world wide trading volume with foreign exchange in 1995 was realized from only a few finance companies in the top five markets : United Kingdom, United States, Japan, Singapore and Hong Kong. 85% of this trade takes place in the top nine (top five plus Switzerland, Germany, France and Australia) alone [2,4].

1.5 Crises

Unfortunately the less established financial markets today are determined by proneness to crises (financial crises in the nineties and later: 1994 Mexico, 1997 East Asia, 1998 Russia, 1999 Brazil [4] and 2001/2002 Argentina.). Short-term transactions are in a high measure responsible for the fluctuations of the rates of exchanges. Volatility favours the development of speculative bubbles and in this way it could give rise to crises. This is especially difficult for developing countries because they react very sensitively on external crises.
A model of financial crises is a four step one by Kindleberger[2]:

First stage:
An external occurence promises opportunities for making profit. The price

for the hopefully profitable thing increases.

Second stage:
This attracts other investors, who want to profit from the increasing prices. The increasing demand causes an increase of the price. The higher the expectation of profit the more money will be invested. If the investors believe in very high profit they raise a loan to invest more in the speculative operation.

Third stage:
The investors come to know that the profit expectation can not be fulfilled. So the profit expectations turn back. The prices increase only slowly or they remain stable.

Last stage:
The prices stagnate and the investors who financed the speculative operation by credits can't pay back the credits. They have to give up all or parts of the securities. The prices of the speculative object decrease. The decreasing prices cause investors to sell their papers panickingly. The worth of the speculative object goes down. So profit expectation turns into losing business and credits can't be payed back. Papers which are used as securities for further credits lose on worth and the banking establishment calls for further securities, repay or they call in the credits. A bank crisis is possible. The credit crisis turns to an insolvency of the inland economy. So the firms are no longer solvent. The financial crisis is now in production sector, and this causes unemployment with its consequences.

World wide, the ILO, in its 1998 World Employment Report, estimated that unemployment increased by 10 million people solely due to the Asian financial crises [3]. Often such crises spread out internationally. Above all short-term capital played an important role in the finance crises of the nineties. Their part on all trades increased about by 300 percent between 1990 and 1995. An important reason for this growth is the removal of controls over turnover of capitals. By means of such controls long-term credits or capital investments could be preferred over short-term investions. Additionally, interests for short-term credits are more favourable than those for long-term credits, because the risk for creditors is greater with long-term credits for developing countries.

1.6 How does currency speculation work ?

If an investor expects the devaluation of a currency A, he will raise a credit in this currency A. The money he gets in this way will be invested in a currency B. When the devaluation of currency A occurs, the investor only has to take a smaller part of his money in currency B to pay back the credit in currency A. The rest of the money in currency B is his profit.

1.7 Herd behaviour

Through hedge funds and other financial organisations which operate internationally the speculation on currencies gets more weight. When a big financial organisation invests a lot of money or calls a lot of money back from a certain transaction, profit or loss expectation from other investors rises. Thus a herd behaviour springs up. A lot of investors will try to get a part of the quickly earned money. Much money will be invested in speculative operations. Sometimes the foreign exchange reserve of the central bank will be spent, if the central bank tries to stabilize the own currency by means of buying. If this method fails, the currency speculation is followed by a devaluation of the currency.

1.8 Trading volume on foreign exchange markets

When in 1971 the USA abandoned the Bretton Woods system of fixed foreign exchange rates and the first transactions with computers came into being, the turnover on the financial markets grew in an abnormal way. From 1970 to 2000 the turnover of the trade with currencies increased from 70 billion to 1.5 trillion US Dollars each trading day. Especially 80% of the 1.5 trillion Dollars are short-term transactions with terms less than seven days, and more than 40% involve round-trips (a purchase operation followed by a resale operation) within two days or less [3]. To estimate the dimensions of speculation we have to take into consideration that annual turnover by 200 trading days in a year reaches 300 trillion US Dollar and this is 40 times the international product and service trade. And 90% of the turnover are speculative short-term transactions and only 10% goes into the production sector [5].

1.9 The Tobin tax

To slow down this process in order that less operations will be speculative and the rates of exchange less fluctuating, James Tobin[2] already suggested in 1972 to impose a tax with a tax rate of 1% affecting all buying and selling currency transactions[3]. Today a tax rate between 0.05% and 0.5% is discussed. The tax Tobin recommended should affect the speculators. The investors try to use smallest price differences even below the 10^{-3} border by foreign rates of exchange. This transactions don't have a real economic meaning. The development of the exchange rates reflect only the hopes of the investors. Supporters of the tax believe that a small tax will make such transactions unprofitable. The rate and the number of such short-term transactions slow down without affecting long-term credits and long-term capital investment. More precisely, they talk about a filter function of the Tobin tax: The tax renders all currency transactions (no matter whether short-term or long-term) more expensive. But because of the different terms the consequences for the different dealing are very different. Short-term transactions are only lucrative when the expectation of the profit is higher than Tobin tax. For example: if Tobin tax rate is 0.5% and one wants to do a transaction for only one week: in this case an annual interest of 52% is necessary to make the transaction profitable. The reduction of short-term transactions will favour long-term transactions. Speculation is reduced and this is hoped to be a prevention for crises. But there is still a fact where Tobin tax is not a method to control. Even the supporters of Tobin tax says that this kind of tax can't minimize speculation operations where 10% to 50% profit is promised. Therefore other controls about turnover of capital should be used. Supporters of the tax named two reasons for introducing Tobin tax: on the one hand the tax would regulate the speculative operations, on the other hand there will be a source of income which is estimated in a two figure billion US Dollars range. Estimates from Felix and Sau in 1996 tell that a

[2]James Tobin was an American economist. Since 1955 he taught at Yale University. In 1981 Tobin became the Nobel Laureate for his papers about monetary theory for governmental financial management [5].

[3]Tobin's proposed tax on international currency transactions, intended to curb speculation, is an extension to foreign exchange markets of Keynes's proposed tax on stock market transactions. Keynes advocated a tax to curb speculation in stock markets. He advocated stronger measures for foreign exchange markets, such as capital controls to defend the autonomy of national stabilization policies. These capital controls were an essential factor to Keynes's wartime proposal for an International Clearing Union [4].

tax rate of 0.25% will reduce the transaction volume less than 33% [5]. With these reasons for Tobin Tax in mind, many people in government or from other organisations like WEED or ATTAC[4] agree to the tax.

2 The model of Cont and Bouchaud [6,7]

We get our results due to combining Tobin tax in two different ways with the microscopic model of Cont and Bouchaud based on percolation theory.
In percolation theory we start to fill the lattice in the way that each site is randomly occupied with probability p and empty with probability $(1-p)$. Neighboring occupied sites form clusters. If a contiguous path of occupied sites connects upper and bottom of the lattice for the first time, the threshold value $p = p_c$ is reached.
In the model presented, the randomly formed clusters are agents who act together, to describe herding behaviour in financial markets. Different clusters reach their decision about buying or selling absolutely randomly and indepently from the other clusters. If there is more supply than demand, the price change decreases proportionally to the difference - otherwise the price change increases. In every iteration, each cluster makes a decision to buy or to sell, each with probability a (which is called the activity). The cluster is inactive in the iteration with probability $(1-2a)$. The maximum value of the activity is 0.5 . We can interpret the activity as a measure for length of the time which we handle in one iteration. If we take the value a close to 0.5, we handle big time steps where nearly all clusters have already made a decision to buy or to sell. Otherwise a small a means that one iteration represents not enough time so that there are only a few clusters who make in this iteration a decision to buy or to sell.
The probability distribution of the price fluctuations shows many smaller and less big fluctuations and the graph is essentially symmetrical. For an activity a close to 0.5 the results at the critical point p_c are similar to a Gaussian curve. For a smaller activity (in particular if we take an a corresponding to intra day time scales) we get heavy tails in the distribution of stock price

[4]On initiative of the french monthly " Le Monde diplomatiqué" come out in june 1998 ATTAC "Association pour une taxation financieres pour l'aide aux citoyens". ATTAC is now represented in 30 countries. A purpose from ATTAC is that policy regulates financial markets and not reverse.

variations in the form of a power law truncated by size effects. Such a behaviour was observed in empirical studies of high frequency market data. Furthermore there exists weak correlations between successive price changes, and strong correlations ("volatility clustering") between successive absolute values of price changes.

The framework of the simulation

- We determine with the complicated algorithm of Hoshen and Kopelman the number n_s of clusters with s agents. Note that if one works with p greater than p_c, one has to ignore the one infinite cluster. This infinite cluster causes only crashes and bubbles.
- We decide randomly if the cluster is active in this iteration.
  - If the cluster is active, we decide by another random number if the cluster would like to buy or to sell an amount which corresponds to the size of the cluster. The return, the difference $\sum_s (n_s^+ * s - n_s^- * s)$ which means especially the difference between demand and supply, is proportional to price change in this time step.
  - If the cluster is not active, it contributes nothing to the return.
- If we have all clusters processed in this way, one iteration is finished and we start again from beginning.

The best agreement with real price fluctuations is found when the concentration is slightly above p_c. One gets the biggest fluctuations when p is equal to p_c. Cont and Bouchaud identified $p = p_c$ therefore with market crashes. There are already different modifications of this model in existence [8].

3 Modifications of the model of Cont and Bouchaud by introducing Tobin tax

In the original Cont-Bouchaud model we have only two parameters: the activity a and the concentration p (the probability to find an occupied site in the lattice. In our simulations we work always at the critical point p_c). The

modified versions of the model used here, has three additional parameters: Tobin tax, Producers and maxwin.
Producers are people who act, even when they not expect to make profit, e.g. to pay a bill.
Because the returns of the Cont-Bouchaud model are often large integers and the Tobin tax is a very small number (less than 1%), we have to normalize the returns. We take two values for maxwin here: 50% and 5%. In the first case, this means that if all clusters in an iteration are active and buying, the return is +50%. Otherwise, if all clusters are active and selling only, the return is -50%. Certainly, not all clusters will buy in the same iteration and not all clusters will sell in the same iteration. We actually get returns between -8% and +8% with the original Cont-Bouchaud model in the case of maxwin= 50%. For the case of maxwin= 5% we get actual returns between -0.8% and +0.8% (see figures 8a, 8b and 9). Note that 80% of the daily speculation trade has taken place because the traders would like to take advantage of profits below the 10^{-3} border. Thus the value maxwin= 5% is rational.

3.1 First modification of the Cont-Bouchaud model

The framework of the simulation

- Because our agents believe that the return of the time step before, which we call r_-, is authoritative for the following return development, we need r_- in this model. In the first time step we took $r_-= 1\%$.
- We determine the number n_s of clusters with s agents with the algorithm of Hoshen and Kopelman.
- We decide randomly if the cluster is active in this iteration.
  - If the cluster is active we have to test, whether the condition (r_- greater than Tobin tax) has been fulfilled or not.
    * If this is the case we decide by another random number, if the cluster would like to buy or to sell an amount which corresponds to the size of the cluster.
      Notice: Both, for buying and selling, we take the condition r_- greater than Tobin tax. In the first case we simulate optimists

(these are people who buy because r_- is greater than Tobin tax and they believe that r_- of next time step will exceed current one) and in the second pessimists (these people sell because r_- is greater than Tobin tax and they believe that r_- in the next time step will be in the same order of magnitude as in the current time step but it will change the sign).
The return, the difference $\sum_s (n_s^+ s - n_s^- s)$ which means the difference between demand and supply, is proportional to the price change in this time step.

  * If the condition is not true we decide through another random number if the cluster is forced to trade because it belongs to the Producers.
    · If the cluster is a Producer we decide randomly if the cluster buys or sells and take it into account for the calculation of the return.
    · If the cluster does not trade it contributes nothing to return or turnover.
- If the cluster is not active it contributes nothing to return or turnover.

- At the end of the iteration we save the return to use it in the next iteration as new r_-. We normalize the returns as described above.
- If we have handled all the clusters in this way, we have finished one time step and we begin the next by redistributing anew the occupied sites.

Note that when there are no Producers and the condition is never fulfilled the return is always zero.

3.2 Second modification of the Cont-Bouchaud model

The framework of the simulation

- We took again $r_-= 1\%$ initially.
- We determine the number n_s of clusters with s agents.

- We decide randomly if the cluster is active in this iteration.
  - If the cluster is active we have to test, whether one of the conditions (r_- greater equal than Tobin tax or r_- less than ($-$ Tobin tax)) has been fulfilled or not.
    * If this is the case we decide by another random number, if the cluster would like to buy or to sell an amount which corresponds to the size of the cluster. The cluster trades in the following two cases: 1. r_- greater than Tobin tax and the cluster will buy (this simulates the behaviour of optimists because they believe that the return will further increase). 2. r_- less than ($-$ Tobin tax) and the cluster will sell (this simulates the behaviour of pessimists because they believe that the return will further decrease). As above, we determine the return by the difference between demand and supply.
- Now for Producers etc. we procede as in the first modification.

We examine the second modification only for RM.

4 Results

To get figures 1-6,11-13 and 16, we took for our calculations an average over about 10000 cluster configurations and a square lattice of length 31. The activity was a=0.4999.
For many of our results we calculate the turnover and sum it up over 500 time steps. With this turnover we determine the profit for government as a function of Tobin tax by multiplication of the turnover with the Tobin tax rate. To get better results we multiply profit for government by 10^8, which corresponds to a summation of turnover about more iterations. The model without Tobin tax can be related to Tobin tax equal zero (which we call "rational model" (RM)) or alternatively to the original Cont-Bouchaud model, where all investors trade even for negative r_- (which we call "similar Cont-Bouchaud model (SCBM)). When we examine y as a function of Tobin tax, the differentiation between these two models leads to an other scale of the y axis. To see this, we show the comparison between these two models

in figures 3 - 6 for the first modification of the Cont-Bouchaud model. We examine the second modification only for RM.

To get the turnover as a function of Tobin tax, we set the turnover we get the way described above for zero Tobin tax to 100%. Then we relate the turnover we get for a positive Tobin tax to the one when we set the Tobin tax to zero.
(Alternatively, we could normalize here again the turnover to that of the original Cont-Bouchaud model.)

4.1 Results of the first modification of the Cont-Bouchaud model

- Turnover as a function of time

  In fig.1a we see that trade will go to zero when there are no Producers. We also see the way the trade volume will decrease with increasing Tobin tax when there are 0.5% Producers. We took maxwin= 5%

  The graph shows from top to bottom:
  The first line shows the turnover we get with the original Cont-Bouchaud model. The next line is a turnover when people only trade, if r_- is greater or equal zero.
  We get the following three by introducing 0.5% Producers with a Tobin tax of zero (empty circles), 0.16% (full circles) and 0.5% (triangles). The next three lines show the development of turnover when nobody has to act, i.e. when there are no Producers, for the same three tax levels.

  What we see if there are no Producers, the turnover approaches zero the faster, the higher Tobin tax is.

- Profit for government

  Fig.1b shows the government profit as a function of the Tobin tax for the RM. We took again maxwin= 5%.
  From top to bottom the graph shows the behaviour when we take the following values for the Producers: 0.5 % , 0.4 % , 0.3 % , 0.2 % and

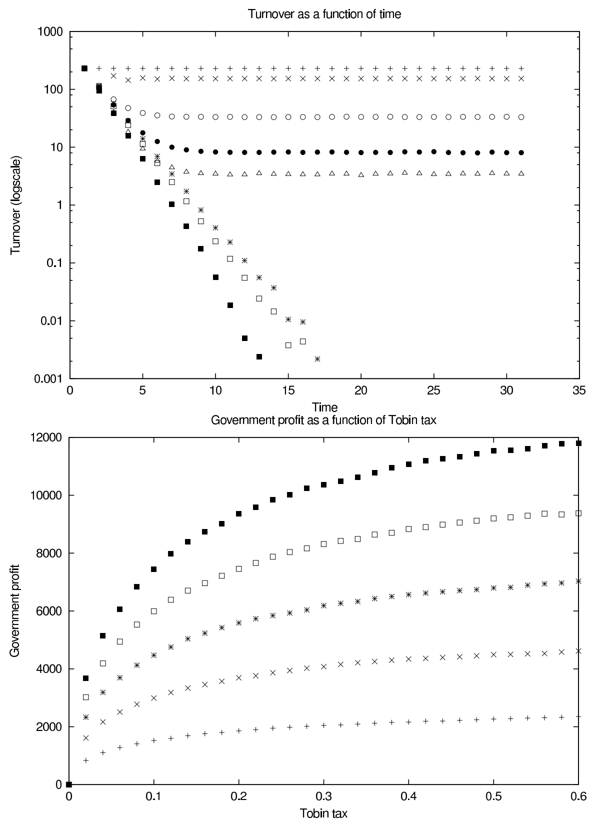

Figure 1: a) Turnover as a function of time. b) Government profit as a function of Tobin tax. From top to bottom: 0.5% , 0.4%, 0.3%, 0.2% and 0.1% Producers.

finally 0.1 %.

In fig.2a we take 1% Producers and fit the first part by the straight line $y = (3983 \pm 630) + (126550 \pm 14585) * x$ (where x represents the Tobin tax rate and y the profit for government)
and the last part by the straight line
$y = (17762 \pm 163) + (10622 \pm 356) * x$.
Both straight lines intersect at $(0.120\% \pm 0.009\%)$ Tobin tax.

Thus if a Tobin tax should be introduced by government, we suggest a value of $\approx 0.12\%$. This is the case where government makes the highest profit from speculators, on condition of returns between $\approx -0.8\%$ and $\approx +0.8\%$ (fig.8b). These returns are reasonable because we have seen above, speculators try to take advantage of profits below the 10^{-3} border. If we take one time step as a week, the extreme return of 0.8% corresponds to an interest calculation of 83.2% per annum. Furthermore we found in our calculations that the value of $\approx 0.12\%$ Tobin tax is independent of the number of Producers as long as there are not more than 5% Producers. Notice that in reality there are far less Producers than 5%, so we can propose a tax of $\approx 0.12\%$.
The straight line with which we fitted the first part of the graph corresponds to ignoring the Tobin tax, in the way that is not dependent on the Producers. The second straight line fitted the part where only the Producers trade.

- Turnover as a function of Tobin tax

  In fig.2b we take 1% Producers and maxwin= 5%. We show the way the turnover will decrease if we introduce a Tobin tax. If we take the proposed value of 0.12%, the turnover decreases by about 87%.

- Profit for government as a function of Tobin tax.
  We will compare RM and SCBM.

  - At first we take maxwin= 5%

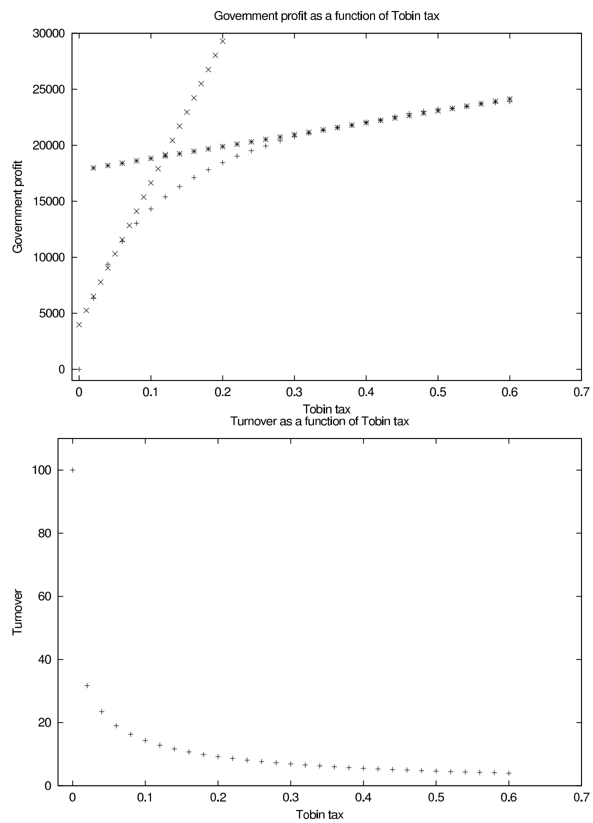

Figure 2: In this figure we take 1% Producers. a) Profit for government as a function of Tobin tax. b) Turnover as a function of Tobin tax.

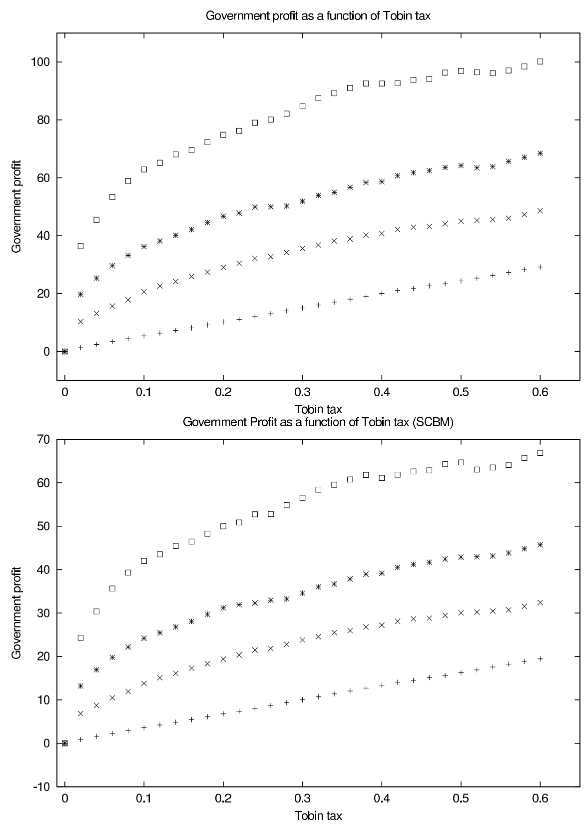

Figure 3: Profit for government as a function of Tobin tax, we take from upper to bottom 0.005%, 0.004%, 0.0035%, 0.003% Producers. a) RM. b) SCBM.

From top to bottom fig.3a (RM) and fig.3b (SCBM) show the curves for 0.005%, 0.004%, 0.0035% and finally 0.003% Producers.

The fig.4a (RM) and fig.4b (SCBM) show that there won't be a difference between the values one should propose for Tobin tax, when we compare both models. We take 0.005% Producers here. The first part of figure 4a is fitted by
$y = (28 \pm 1) + (425 \pm 17) * x$

The second straight line which is used to fit the second part is:
$y = (71 \pm 2) + (50 \pm 4)$*x

The first part of figure 4b is fitted by
$y = (19 \pm 1) + (281 \pm 11) * x$

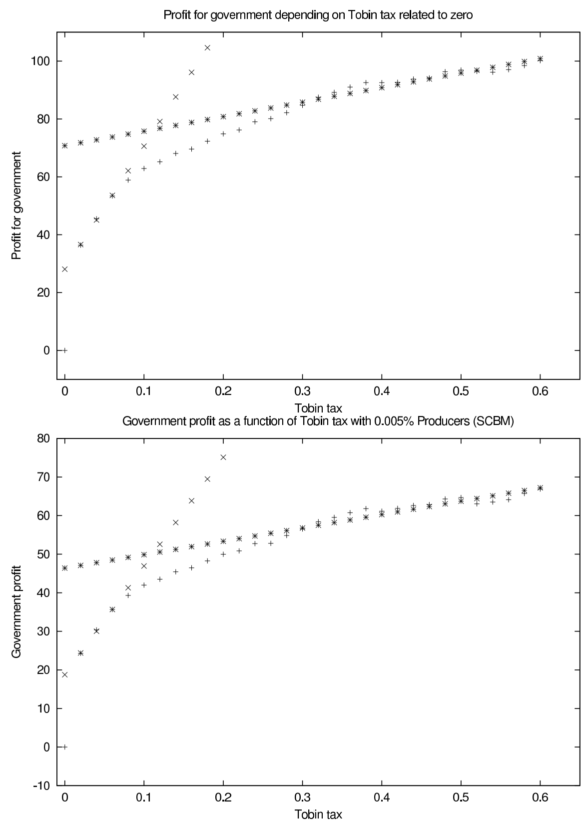

Figure 4: Profit for government as a function of Tobin tax, we take 0.005% Producers and maxwin= 5%. a) RM. b) SCBM.

The second straight line which is used to fit the second part is:
$y = (46 \pm 1) + (35 \pm 3) * x$

Again we can state that the convenient Tobin tax is $(0.113\% \pm 0.013\%)$ (which corresponds to the value we got above within the error margin) when we relate to zero (RM).
And when we take the model similar to Cont-Bouchaud (SCBM), we get for government a convenient Tobin tax of $(0.118\% \pm 0.008\%)$.

– Now we take maxwin=50%

The returns one gets now are actually between -8% and $+8\%$.

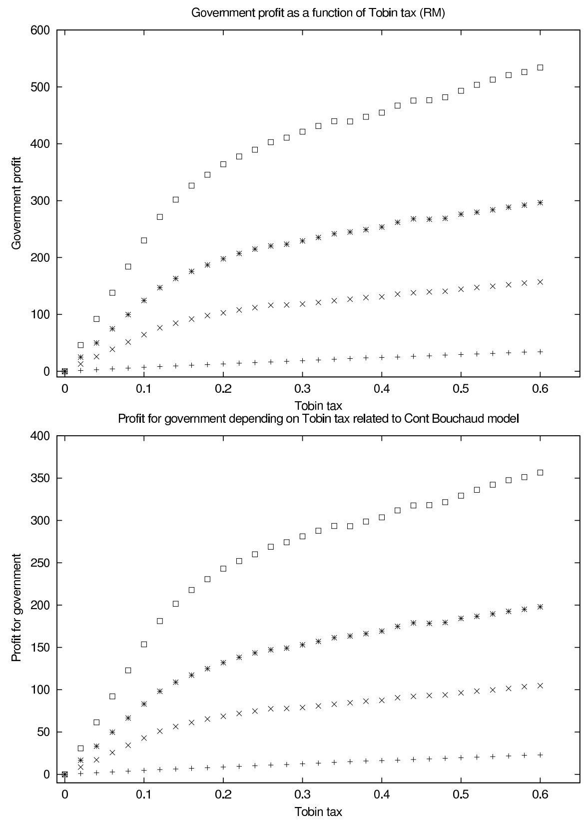

Figure 5: Profit for government as a function of Tobin tax, we take 0.005%, 0.004%, 0.0035% and 0.003% Producers and maxwin= 50%. a) RM. b) SCBM.

From top to bottom in fig.5a and fig.5b are curves for 0.005%, 0.004%, 0.0035% and finally 0.003% Producers.

We see in fig.5a and fig.5b that there is an expected difference for proposing a Tobin tax depending on the value of maxwin.

The first part of fig.6a is fitted by
$y = (0 \pm 0) + (2301 \pm 0) * x$

The second straight line which is used to fit the second part is:
$y = (303 \pm 3) + (385 \pm 7)$*x

Both straight lines intersect at (0.158 ± 0.002)% Tobin tax.

The first part of fig. 6b is fitted by

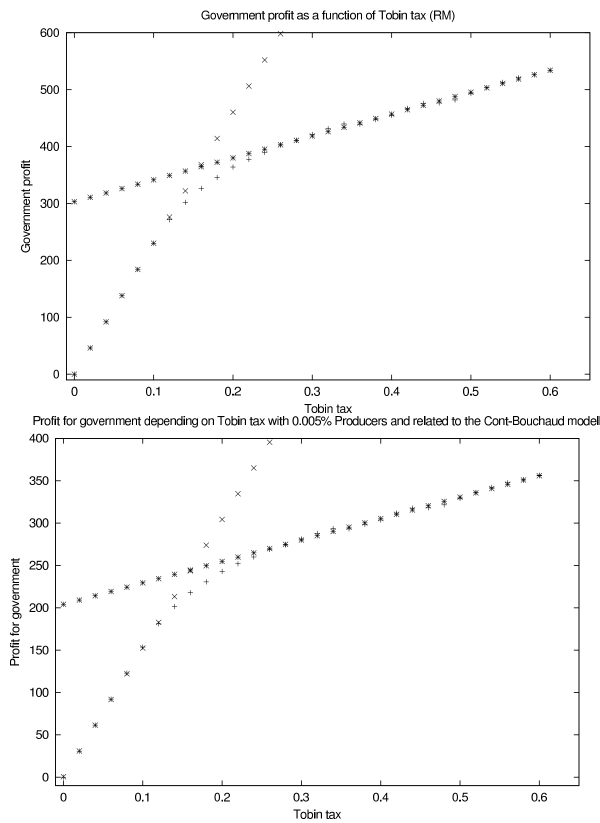

Figure 6: Profit for government as a function of Tobin tax, we take 0.005% Producers and maxwin=50%. a) RM. b) SCBM.

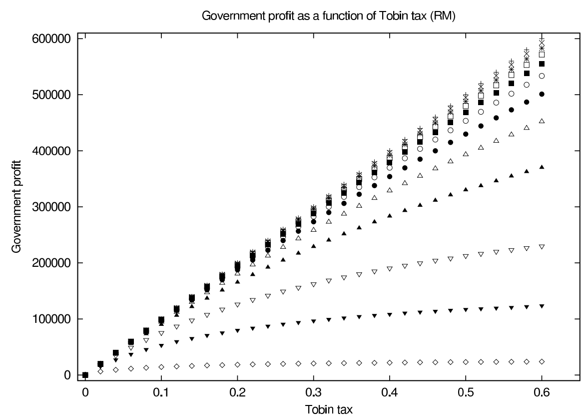

Figure 7: Profit for government as a function of Tobin tax with increasing Producers,maxwin=5%

$y = (0.55 \pm 0.69) + (1519 \pm 10) * x$
The second straight line which is used to fit the second part is:
$y = (204 \pm 2) + (253 \pm 5) * x$
Here both straight lines intersect at (0.161 ± 0.004)% Tobin tax.

Both values we get here for Tobin tax agree within the margin errors.
Result here: If we take maxwin as 50% we should propose a Tobin tax of 0.16%.

- Profit for government as a function of Tobin tax with increasing Producers.

Here in fig.7 we have from top to bottom 100%, 90%, 80%, 70%, 60%, 50%, 40%, 30%, 20%, 10%, 5% and finally 1%. One can see that we get straight lines as long as there are 50% Producers . These straight lines show that there is an insensitivity against the Tobin tax.
But in real currency markets there are not so many Producers. The number of Producers will converge to a very small number. But it does not converge to zero because there are people who must sell because they need money.
We get the following straight lines from top to bottom:

Straight lines which fit profit as a function of Tobin tax	
Producers	Straight line
100% $\Leftrightarrow$ Cont-Bouchaud	$y = 0$
90%	$y = (987570 \pm 607) * x - (1168 \pm 212)$
80%	$y = (971500 \pm 1335) * x - (2615 \pm 466)$
70%	$y = (952320 \pm 2297) * x - (4493 \pm 802)$
60%	$y = (924940 \pm 3561) * x - (6987 \pm 1243)$
50%	$y = (888740 \pm 5244) * x - (10369 \pm 1832)$
70%	$y = (833460 \pm 7571) * x - (15207 \pm 2644)$

- Return as a function of time

  In the following Figs.8 and 9 the "+" sign represents the returns in the original Cont-Bouchaud model, the "x" sign represents the returns when r_- should be greater or equal to zero, " * " shows the behaviour when a Tobin tax of 0.01% is introduced and the empty squares stand for returns when a tax of 0.16% is introduced.
  We took a square lattice of length 31 and an average about 100 cluster configurations.
  In fig.8a (maxwin= 50%) and fig.8b (maxwin= 5%) we took 1% Producers. In fig.9 we took only 0.005% Producers (maxwin= 5%).

- The Return Histogram

  The deviation of the heavy tails from the Gaussian will be characterized by a significant excess kurtosis which is defined by

$$\kappa = \frac{\mu_4}{\sigma^4} - 3 \quad , \tag{1}$$

  where μ_4 is the fourth central moment and σ the standard deviation of the returns. κ should be zero for a normal distribution but it ranges between 2 and 50 for daily returns and is even higher for intra day data. The fat tails correspond to large fluctuations in returns.
  For the following calculations we took a square lattice of length 101. We took an average of about 100 cluster configurations and made 100000 iterations.
  Here are the prices not normalized and we take 1% Producers.

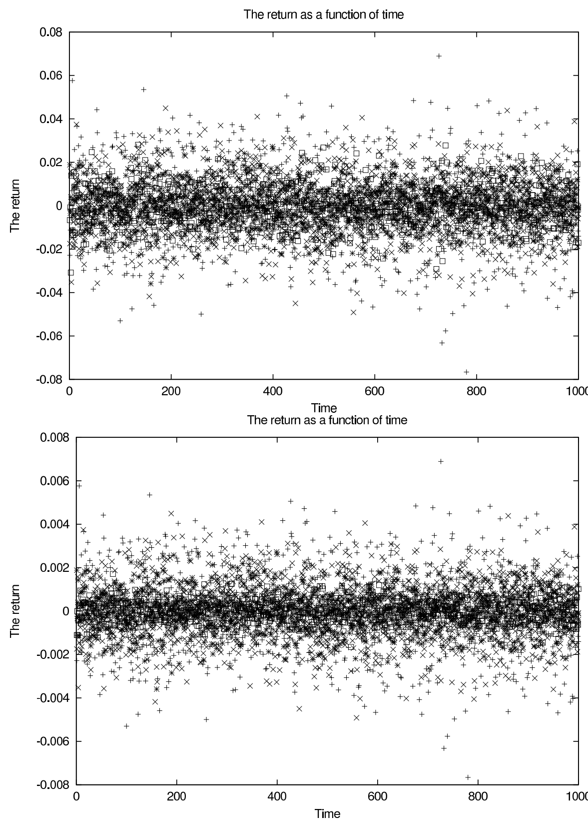

Figure 8: Return as a function of time, 1% Producers. a) maxwin=50%, b) maxwin=5%

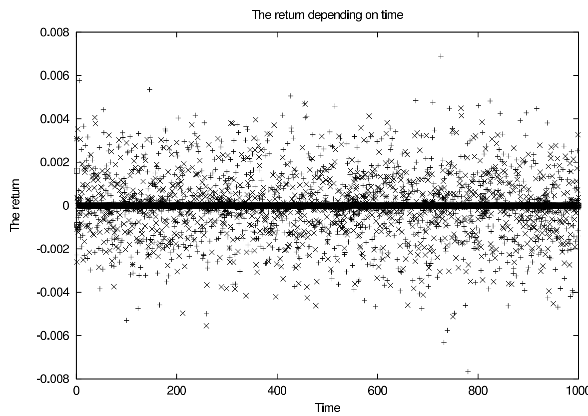

Figure 9: Return as a function of time, maxwin=5%, Producers= 0.005%

Figure 17 shows from outside to inside the Gaussian for the original Cont-Bouchaud model for which we get a kurtosis of $\kappa = 0.58$ here (probably this is due to the small lattice). The next interior curve represents the case when people act when r_- is greater or equal zero ($\kappa = 3.71$). What follows is the curve where Tobin tax amounts 0.01% ($\kappa = 8.38$) and for the last one a Tobin tax of 0.16% ($\kappa = 56.76$) is assumed. In figure 16 we have add the following two cases: The Cont-Bouchaud model with activity a=10% and a=1%. The outer one of the two added curves corresponds to activity a=10% ($\kappa = 4.96$) and the other one to a=1% ($\kappa = 54.72$).
From the picture we get the results that the effect of "convergence" to zero is the greater the greater Tobin tax is and the "convergence" to zero is for a $\rightarrow$ 1% not so drastic as for Tobin tax $\rightarrow$ 0.16%.

- Number of speculating and trading investors

For Fig.11, we sum up the number of investors who are former speculators (+) or still active speculators (x) over about 500 time steps for different values of Tobin tax rate. We define former speculators in the following way: The former speculators are those people who don't trade because they do not fulfill the condition r_- greater than Tobin tax. Here is maxwin= 5% and Producers= 1%. Fig.11 shows the result for the RM. We have seen above that a Tobin tax rate of 0.12%

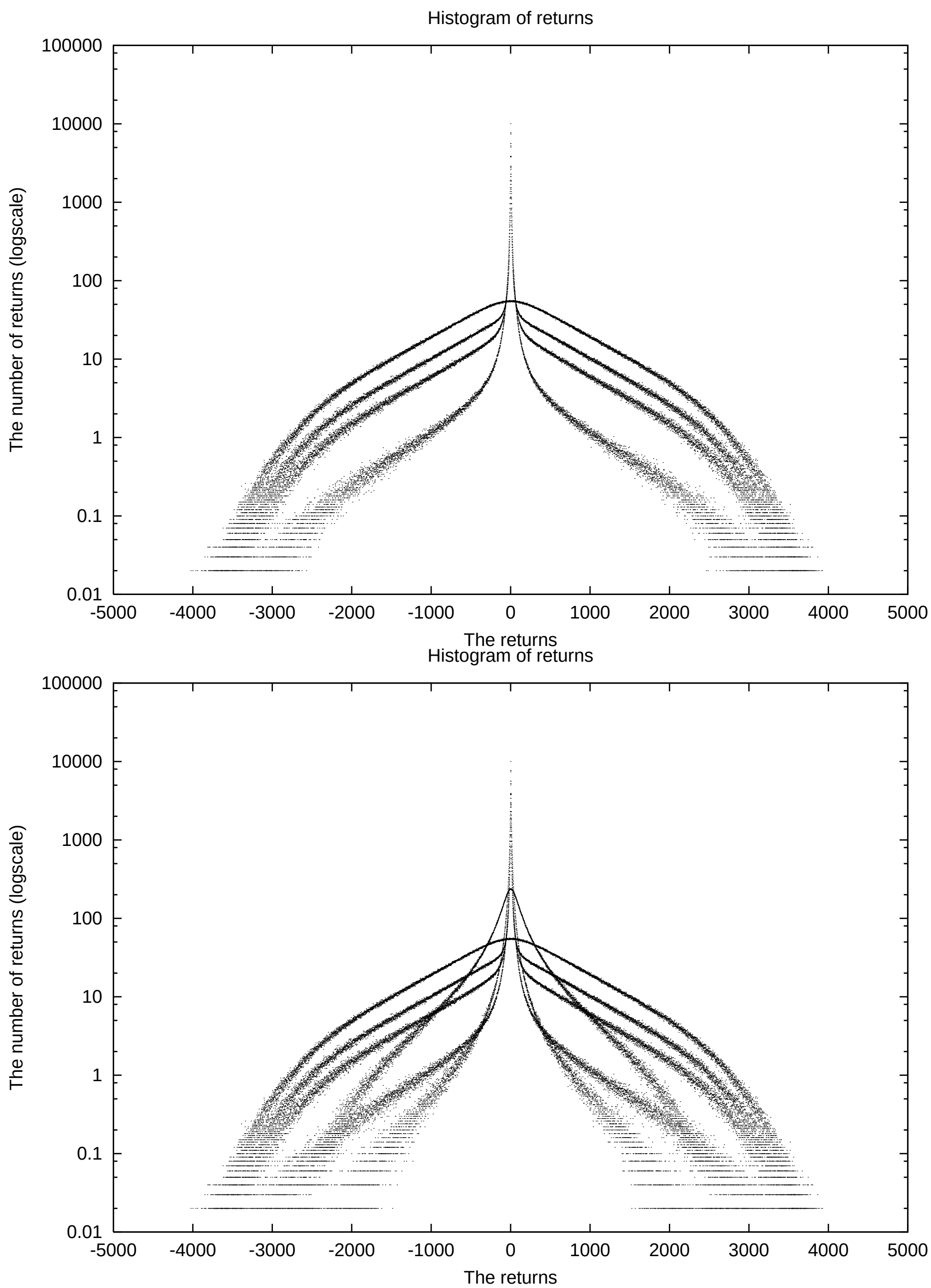

Figure 10: a)Histogram of returns. b) Histogram of returns compared with different activities.

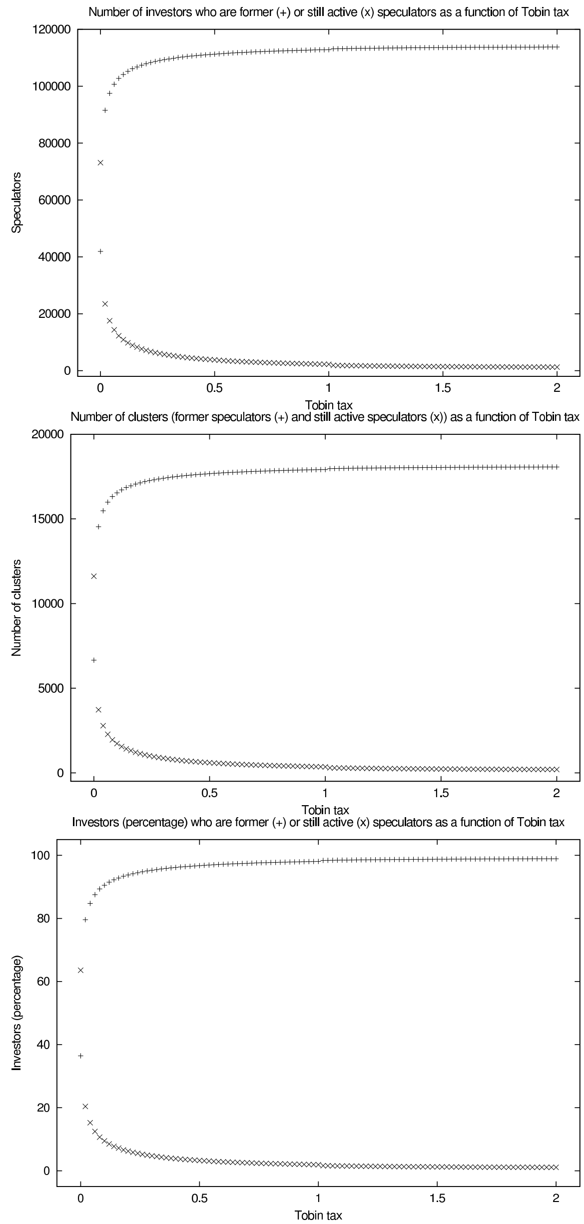

Figure 11: RM. a) Number of former speculators (+) and still active speculators (x). b) Number of clusters (former speculators (+) and still active speculators (x). c) Former speculators (+) and still active speculators (x) (percentage).

is convenient for government. But you can see that we have here 91% former speculators and only 9% still active speculators.
The SCBM gives nearly the same result except the fact that when Tobin tax is equal zero there are no former speculators and all available investors are active speculators.
The sum about number of clusters corresponding to the sum about the number of investors lies between 18265 and 18275.
If we take a look on the number of investors who are former or active speculators as a function of time, we get straight lines.

Straight lines which fit the number of former speculators as a function of time	
Condition to trade	Straight line
Cont-Bouchaud	$y = 0$
$r_- \geq 0\%$	$y = (83.833 \pm 0.002) * x - (66.062 \pm 0.466)$
$r_- \geq 0.01\%$	$y = (175.362 \pm 0.003) * x - (268.98 \pm 0.870)$
$r_- \geq 0.16\%$	$y = (208.527 \pm 0.005) * x - (379.22 \pm 1.587)$

Straight lines which fit the number of still active speculators as a function of time	
Condition to trade	Straight line
Cont-Bouchaud	$y = (229.642 \pm 0.001) + (3.305 \pm 0.429)$
$r_- \geq 0\%$	$y = (145.774 \pm 0.003) * x + (68.755 \pm 0.764)$
$r_- \geq 0.01\%$	$y = (54.265 \pm 0.003) * x + (266.055 \pm 0.777)$
$r_- \geq 0.16\%$	$y = (21.029 \pm 0.003) * x - (359.080 \pm 0.926)$

For every value of Tobin tax we have an average cluster size of (6.42 ± 0.02) and the "infinite" cluster we ignored because it causes only crashes and bubbles has an average size of (311.38 ± 0.97) .

4.2 Results of the second modification of the Cont-Bouchaud model

With this modification we did in generally the same calculations as described above but only for RM.

- Turnover as a function of time

  In fig.12a we see that trade will go to zero when there are no Producers. We also see the way the trade volume will decrease with increasing Tobin tax when there are 0.5% Producers. We took maxwin= 5%

The graph shows from top to bottom:

The first line shows the turnover we get with 0% Tobin tax and 0.5% Producers. The next line is a turnover with 0% Tobin tax and 0% Producers.
We get the next following two lines which does not present a decrease of turnover after by about 100 time steps with 0.5% and 1% Tobin tax and in both cases 0.5% Producers. The next two lines describe the behaviour for 0% Producers for 0.5% Tobin tax and 1% Tobin tax. What we see if there are no Producers, the turnover approaches zero the faster, the higher Tobin tax is, but the decay is not so drastic as we have seen with the first modification of the Cont-Bouchaud model.

- Profit for government

  Fig.12b shows the profit for government as a function of the Tobin tax. From top to bottom the graph shows the behaviour when we take the following values for the Producers: 15%, 11%, 10%, 8%, 7%, 6%, 5%, 4%, 3%, 2%, 1%, 0.1% and finally 0%.
  We see here that the maximum goes to higher Tobin tax rates for an increasing number of Producers. We took here maxwin= 5%. When we took maxwin= 50% the maximum of profit for government is above 1.5% Tobin tax.

- Tobin tax to get maximal profit

  Fig.13a shows the Tobin tax value one should take to get maximal income for government (Ttp) as a function of the Producers (P). The curve (+) is especially well fitted between 0% and 2% Producers (this corresponds to reality values for Producers) by

$$Ttp = \sqrt{\frac{P}{25}} + \frac{3}{10}$$

  .

- Turnover as a function of Tobin tax

  In fig.13b we take 1% Producers and maxwin= 5%. We show the way the turnover will decrease if we introduce a Tobin Tax. The "+" sign

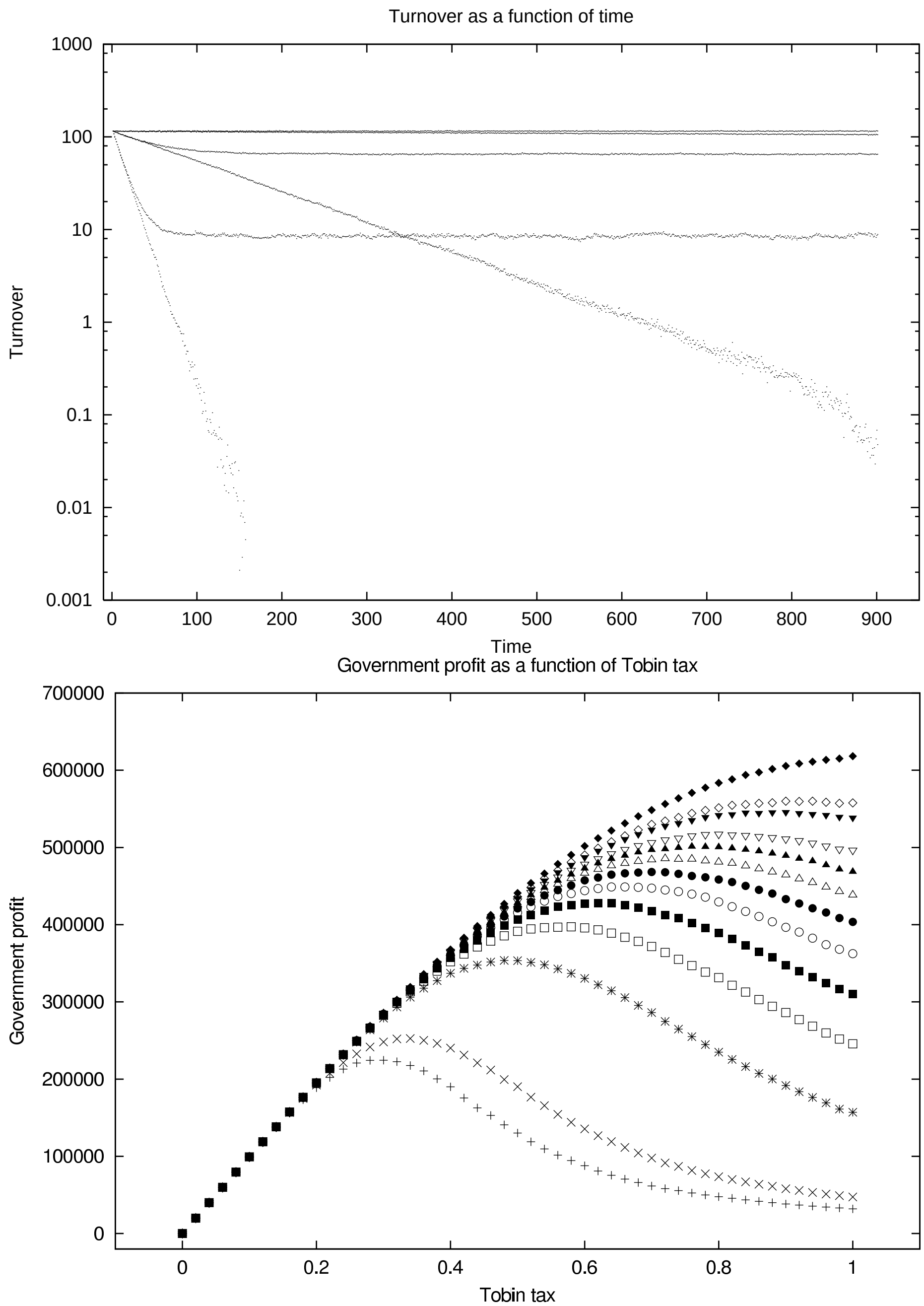

Figure 12: a) Turnover as a function of time. b) Profit for government as a function of Tobin tax. From top to bottom: 15%, 11%, 10%, 8%, 7%, 6%, 5%, 4%, 3%, 2%, 1%, 0.1% and finally 0% Producers.

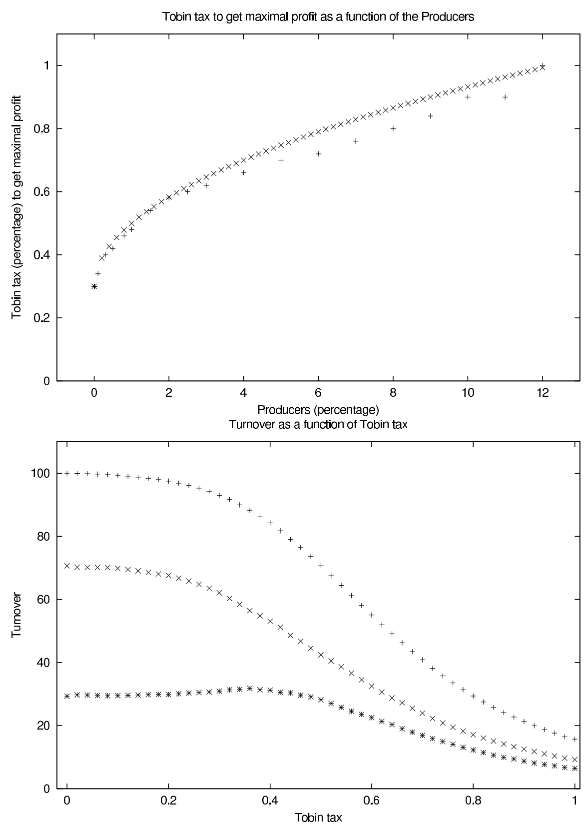

Figure 13: a) Tobin tax to get maximal profit as a function of Producers. b) Turnover as a function of Tobin tax with 1% Producers.

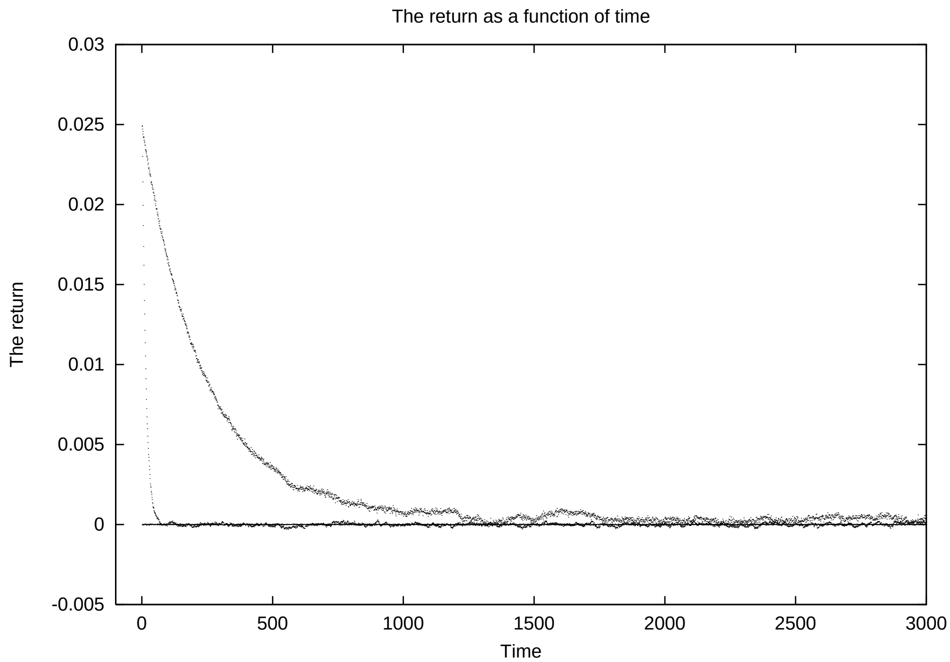

Figure 14: The return as a function of time.

is the turnover we get by buying and selling, “x” presents just the turnover by buying and “*” shows the turnover we get only by selling. To get the maximal profit we have to take a Tobin tax of 0.48% under condition of 1% Producers (we call this value T_{max}. T_{max} is Ttp for one percent Producers). We see here that we have there still a turnover of 73%.

- Return as a function of time

  We calculate the return as a function of time as described in the first modification of the Cont-Bouchaud model. We take 1% Producers and 5% maxwin. The first line of fig.14 shows the behaviour for 0% Tobin tax, the next line for 1% Tobin tax and the line which fluctuate about zero (even for t equal 1) is for 0.48% Tobin tax. We see here and with the following tabular that the prices will less fluctuate at the value of Tobin tax where government profit has a maximum. We got the following tabular by maxwin= 5%. Tt means “Tobin tax”, STFZ is

the first time step where the return starts to fluctuate about zero, sd means "standard deviation" and we find in the last column the value of r_- at t=1 which we call r_-1.

Tt	actual returns	STFZ	average	sd	r_-1
0	(−0.04%)-0.12%	1000	−4.1E(−2)	2.6E(−2)	2.49%
0.01	(−0.1%)-0.08%	1000	−1.7E(−3)	2.7E(−2)	2.49%
0.16	(−0.09%)-0.09%	1000	1.4E(−3)	2.7E(−2)	2.49%
0.2	(−0.09%)-0.11%	1000	−7.9E(−3)	2.9E(−2)	2.49%
0.4	(−0.09%)-0.11%	1000	−4.7E(−3)	2.4E(−2)	2.46%
0.48	(−0.007%)-0.007%	0	−1.7E(−4)	1.9E(−3)	0.001%
0.5	(−0.007%)-0.007%	0	−1.7E(−4)	1.9E(−3)	0.001%
1.0	(−0.03%)-0.03%	70	−2.7E(−4)	9.2E(−3)	2.49%

The returns as a function of time have a minimum of fluctuations for $T = T_{max}$. This shows that one get both, on the one hand maximal government profit and on the other hand a minimum of return fluctuations which is important to be able to reduce speculation.

The following tabular we get for maxwin= 50%. Remember that in this case the maximum of government profit is above 1.5% Tobin tax.

Tt	actual returns	STFZ	average	sd	r_-1
0	(−0.3%)-1.2%	1000	4.1E(−1)	2.6E(−1)	24.9%
0.5	(−0.8%)-0.8%	1000	−4.3E(−2)	2.4E(−1)	24.9%

You see that here the return fluctuations are much higher than for maxwin =5%.

- The Return Histogram

Fig.15a: shows from outside to inside the behaviour for 0% Tobin tax and 0.5% Tobin tax for an activity of a= 1%. Fig.15b shows the price histogram for an activity of 50% and a Tobin tax of 1.8%. Unfortunately we have not a realistic price histogram for an activity of 50% and a Tobin tax below 1.8%.

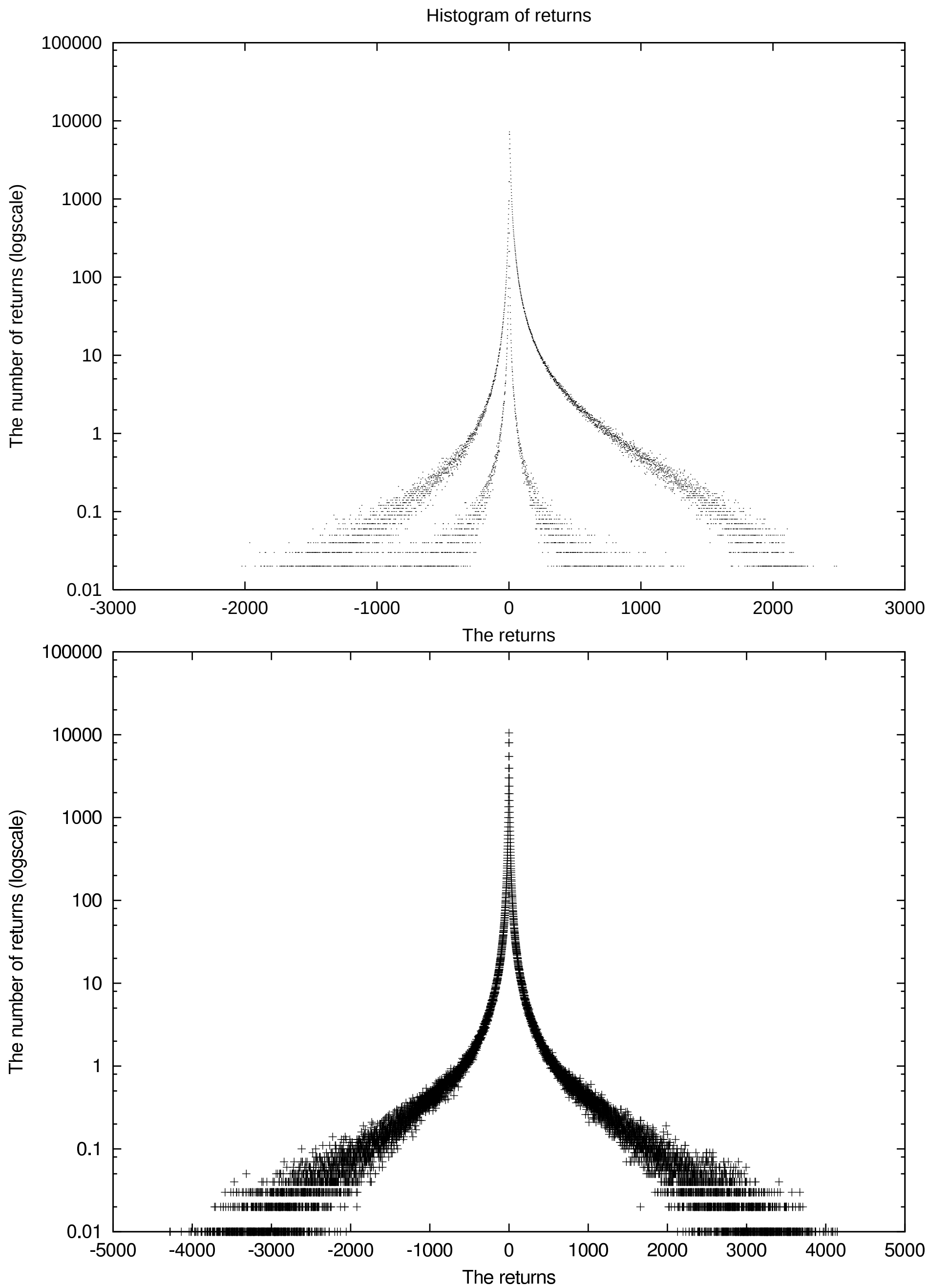

Figure 15: a) Histogram of returns with a=1%.

Kurtosis		
activity	Tobin tax	Kurtosis
50%	1.8	160
50%	2	170
50%	10	109
1%	0	103
1%	0.5	5130
1%	1	5271
1%	10	5604

- Number of former speculating and still active speculating investors

  For Fig.16, we sum up the number of investors who are former speculators (+) or still active speculators (x) about 500 time steps for different values of Tobin tax rate. We define former speculators in the following way: The former speculators are those people who don't trade because they do not fulfill one of the conditions r_- greater than Tobin tax or r_- less than ($-$ Tobin tax). Here is maxwin= 5% and Producers= 1%. The curves intersect at 0.53% Tobin tax. At T_{max} (0.48% Tobin tax) we have here 42% former speculators and 58% still active speculators.

Summary

In conclusion, the first modification of the Cont-Bouchaud model towards an introduction of a Tobin tax does not lead to the desired maximum value for profit as a function of Tobin tax rate. Instead, it discourages most speculators (96%) if it becomes of the order of 0.5%.
The differentiation between these two models (RM and SCBM) does not lead to different values one should propose for a Tobin tax rate. But when we multiply the returns of Cont-Bouchaud model by 10, the Tobin tax rate needs to be multiplied by 1.33.

Results about Tobin tax which one should propose			
maxwin	return	tax rate (RM)	tax rate (SCBM)
5%	±0.8%	0.12%	0.12%
50%	±8%	0.16%	0.16%

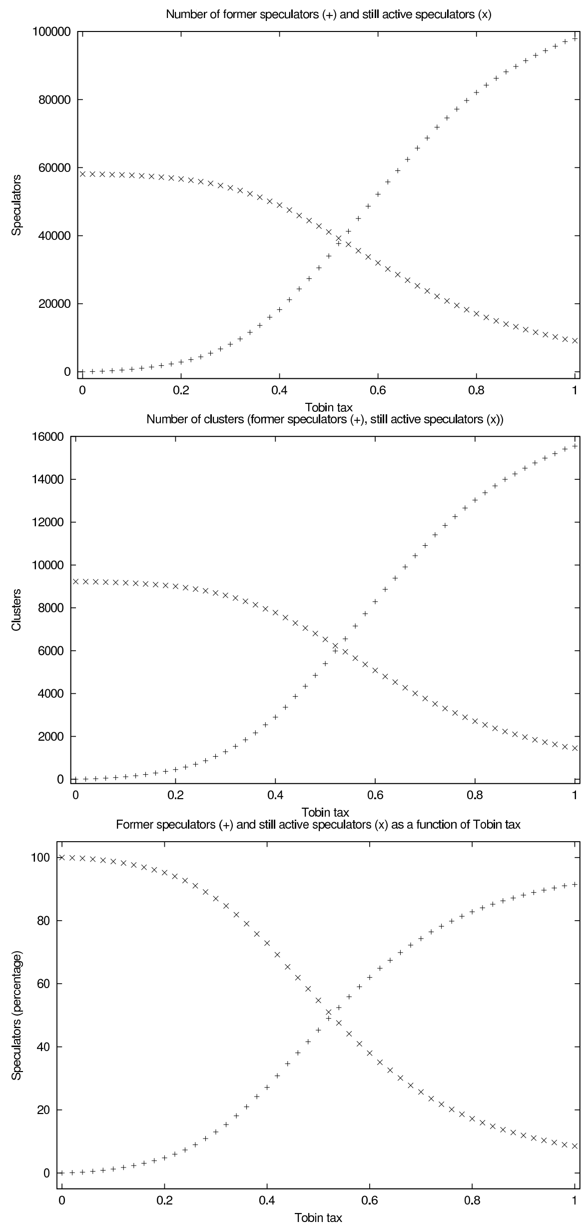

Figure 16: a) Number of former speculators (+) and still active speculators (x). b) Number of clusters (former speculators (+) and still active speculators (x)). c) Number of former speculators or still active speculators (percentage).

Kurtosis		
activity	Tobin tax or model	Kurtosis
50%	original Cont-Bouchaud model	0.58
10%	original Cont-Bouchaud model	4.96
1%	original Cont-Bouchaud model	54.72
50%	$r_- \geq 0\%$	3.71
50%	$r_- \geq 0.01\%$	8.38
50%	$r_- \geq 0.16\%$	56.76

The second modification leads to a desired maximum for government profit, sufficient turnover and damped return oscillations but not to a realistic return histogram for maximal activity and a Tobin tax below 1.8%. On the other hand it is a further success of this modification that when we work at T_{max} we discourage 42% of the speculators.

Acknowledgements

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